Ph125c

Spring 2006

QUANTUM MECHANICS

Problem Set 7

Due in class Wednesday, 17 May 2006

Reading assignment: Sections 1-3 (up to spin waves and magnons) in Chapter 21 in the *old* (2nd) edition of Merzbacher's quantum mechanics book (handed out in class and available in Lauritsen 264). This week's problem set deals with spin waves. You'll have to do the reading carefully to get through the problem set. If you can get through this problem set, then you really understand the theory of spin waves, some pretty sophisticated quantum mechanics.

1. Show that the function $\varphi(\kappa)$ that appears in the magnon dispersion relation is an even function. Show that in the low-momentum limit the energy-momentum relation for a magnon is similar to that for a nonrelativistic particle. Identify the effective mass of a magnon.

2. Derive the spin-wave dispersion relation for a lattice in which each spin interacts only with its nearest neighbor.

3. Prove that the state
$$\frac{1}{\sqrt{2\pi}}\sum_j e^{i\kappa j}a^\dagger_{1/2,j}a_{-1/2,j}|0\rangle,$$
is an eigenstate of the *exact* Heisenberg Hamiltonian, and calculate the corresponding eigenvalue if the spin lattice is is one-dimensional and all spins are equivalent.

4. From the Heisenberg Hamiltonian, derive the equations of motion for the spin operators $\vec{S}_k$ in the Heisenberg picture. Assuming that the operators may be replaced approximately by classical vectors and that, as is the case near the ground state, each $\vec{S}_k(t)$ differs only slightly from a fixed common vector $\vec{S}$, obtain linear equations of motion for the *spin deviation vectors*, $\vec{s}_k(t) = \vec{S}_k(t) - \vec{S}$, if $|\vec{s}_k(t)| \ll |\vec{S}|$. Show that the equations of motion for the spin deviation vectors describe the propagation of spin waves through a periodic lattice, each vector $\vec{s}_k(t)$ precessing about $\vec{S}$. For a one-dimensional latticve in which all spins are equivalent, show that the relation between the precession frequency ω and teh wavenumber κ/d is given by the spin-wave dispersion formula, provided that the magnon energy is identified as $E_\kappa = \hbar\omega$.