

Ph125c
Spring 2006

QUANTUM MECHANICS

Problem Set 8

Turn in to a TA by Friday, 26 May 2006

Reading assignment: Section 6 (on pairing interactions) in Chapter 21 in the *old* (2nd) edition of Merzbacher's quantum mechanics book (handed out in class and available in Lauritsen 264).

1. In class we used the Bethe-Goldstone equation to show that a Fermi sea guarantees the existence of a bound state for two electrons at the top of the Fermi sea, even for an arbitrarily small attractive interaction. Use the same approach to show that if there is no Fermi sea (i.e., if the Fermi momentum $\hbar k_F = 0$), then there is not necessarily a bound state.
2. Calculate the mean-square radius r of a Cooper pair under the approximation where the momentum-space matrix elements of the interaction are $V_{\vec{k},\vec{k}'} = -V/L^3$ for $\hbar^2 k^2/2m < E_F + \hbar\omega_D$ and $\hbar^2 (k')^2/2m < E_F + \hbar\omega_D$ and $V_{\vec{k},\vec{k}'} = 0$ otherwise (i.e., that the interaction matrix elements are nonzero and constant only in the narrow region above the Fermi surface). You should be able to write your answer in terms of \hbar , the Fermi velocity v_F , and the binding energy E of the Cooper pair.
3. Calculate the variance of the total number of particles in the BCS ground state in terms of v_m and u_m .