Quantum Field Theory (171.702), Spring 2023

Problem Set 1

Due: 27 February 2023

The calculation of the beta function for QCD is one of the most significant in the history of particle theory. It resulted in Nobel prizes for David Gross, Frank Wilczek, and David Politzer. Now its a homework problem—actually, two.

1. Go through the background-field-method derivation of the Yang-Mills beta function in Tong's notes, and fill in the details. This is discussed on pages 68–73 of Tong's notes. The algrebra through the end of p. 70 should be straightforward. Be sure, though, that you go through the steps required to evaluate the traces (of $\log \Delta_{\text{gauge}}$ and $\log \Delta_{\text{ghost}}$—that is the hard part.

2. The QCD beta function was originally derived perturbatively, with Feynman diagrams, in analogy with the derivation of the running of the coupling constant for QED that we went through last semester. Re-derive the QCD beta function this way. Its a long process, and you should feel free to use the original papers or the presentations in other QFT books (e.g., Srednicki's book).

3. Consider a Wilson loop in Euclidean spacetime that corresponds to an electron-positron pair produced at some time $T = 0$ and are then separated to a large distance R for a time $T \gg R$ at which time the two particles are brought together and then annihilate. Show by explicit calculation that the vacuum expectation value for this loop agrees with what you'd expect for two particles with an attractive Coulomb force.