**Ay123**

**Fall 2005**

## STELLAR STRUCTURE AND EVOLUTION

## FINAL EXAM

Due Friday, December 9, 2005

This is a closed book exam. No notes or consultation. An unprogrammed handheld calculator is ok, if you need it. Please take three contiguous hours in a quiet isolated location, and do not look at the problems until you begin. Some possibly useful constants are provided at the back.

**Instructions:** Do problem 1 [50%]. Then do one of problems 2 or 3 [25%]. And then choose any two of problems 4, 5, or 6 [25%].

1.

a. Write down the four stellar structure equations for a star of mass $M$.

b. Specify the boundary conditions, and explain why the number of boundary conditions you use is correct.

c. Write down the order of magnitude equivalent of these equations.

d. Assume that the opacity is that of Thomson, and take care in keeping proper account of $\mu$ and $\mu_e$ in your equations. Derive an order-of-magnitude estimate for the luminosity of a star as function of its mass. Show that it is independent of the reaction-rate function $\epsilon$.

e. What is the value of $\mu$ and $\mu_e$ for pure hydrogen, and for pure helium. Explain how the luminosity of the Sun changed in the past.

f. Which of these (order of magnitude) equations should be used for pre-main sequence stars, and which is not valid? Discuss both convective and radiative stars in your answer.

g. Derive the path of a pre-main sequence star once it leaves the vertical Hayashi line and approaches the main sequence. You only need to derive the slope of the curve in the HR or ($T_{eff}$,$L$) plain. Use Kramer's opacity since the interior arrive to the MS temperature just at the end, it is mostly colder and Kramer's opacity applies even for solar mass stars.

h. What is the age of a solar-mass star located within that line as function of its luminosity? Derive the functional dependence and then scale your answer using your knowledge about the Sun.

2. For a star of a given mass $M$, what is the total energy output over its lifetime? Assume that it has an efficient momentum driven wind, and derive an expression for the *fraction* of ejected mass over the lifetime of the star. You should be able to express your answer in terms of the escape velocity from the surface of the star (and physical constants). Argue that the escape velocity, and therefore the ejected fraction, are not a strong function of the stellar mass.

3. Assume that the temperature required for significant nuclear reactions is $kT = 1\ keV$. Use that to derive the minimum stellar mass, or the star/brown dwarf mass limit. Express your answer as function of fundamental constants and kT. Substitute numbers and compare to the real limit.

4. Estimate $E_0$, the most probable fusion energy, for the $p + p \rightarrow d + e^+ + \nu_e$ reaction in the Sun.

5. A star is rotating with an equatorial velocity of 50 km/sec. This rotation broadens its lines. Estimate the half widths of lines due to this rotation. Compare this half widt with the thermal Doppler half widths of an $Fe^+$ line in an A0 V star. Describe the difference in the profiles of thermally and rotationally broadened lines.

6. Compare the total energy released by a 15 $M_\odot$ star during (a) its time on the main sequence, (b) its post-main-sequence evolution, and (c) its supernova explosion.

Proton mass: $m_p = 1.67 \times 10^{-24}$ g
Electron mass: $m_e = 9.11 \times 10^{-28}$ g
Planck's constant: $h = 6.6 \times 10^{-27}$ erg-s (and $\hbar = h/2\pi$)
Solar mass: $M_\odot = 1.99 \times 10^{33}$ g
Solar luminosity: $L_\odot = 3.85 \times 10^{33}$ erg/s
Boltzmann constant: $k_B = 1.38 \times 10^{-16}$ erg/K
Stefan-Boltzmann constant: $\sigma = \pi^2 k_B^4/60\,\hbar^3 c^2 = 5.67 \times 10^{-5}$ erg/cm$^2$/K$^4$
Energy: 1 eV$= 1.6 \times 10^{-12}$ erg