Ay123

Fall 2005

STELLAR STRUCTURE AND EVOLUTION

Problem Set 4

Due Tuesday, October 25, 2005

2. Estimate the maximum angular frequency for gravity waves in the Earth's atmosphere. To do so, treat the atmosphere as an isothermal atmosphere.

2. White dwarfs are observed to undergo oscillations with periods (very roughly) of ~ 500 sec. Using typical white-dwarf numbers (e.g., carbon interior, mass $M = 0.6\, M_\odot$ and radius $R = 0.01\, R_\odot$), to show that the the Brunt-Vaisala frequency in the outer envelope of the white dwarf is roughly consistent with the observed g-mode frequency. How does this result depend on the internal temperature?

3. *Convection in a white dwarf envelope.* Consider a white dwarf of $0.6\, M_\odot$, radius $0.01\, R_\odot$ and effective temperature of 12,000 K. A model envelope, assumed to be pure hydrogen with mixing length $l_m = 2H_P$ (i.e., $\alpha = 2$) is found to pass through the following point in the ρ-T plane: $\log T = 4.29$ and $\log \rho = -5.51$ (in cgs units). At that point in the star,
   a. Calculate the hydrogen ionization fraction.
   b. Calculate the pressure p. Use this pressure and the equation of hydrostatic equilibrium to calculate the total mass (in units of $M_\odot$) of the envelope above this point.
   c. Calculate $\nabla_{\rm ad}$.
   d. Do a google search to find the OPAL opacity tables (should be somewhere at ornl.gov). Find the opacity for pure hydrogen at this temperature and density (this might take some poking around on the OPAL site, but not too much. Keep in mind that the tables give opacities as a function of $\log T$ and $\log R$, where $R = \rho(T/10^6\, rmK)^{-3}$.)
   e. Use the equation of radiative equilibrium to calculate $\nabla_{\rm rad}$. Show that at this point in the star, (at least part of) the flux must be carried by convection.
   f. Calculate the pressure scale height H_P at this zone, and show that the Rosseland optical depth is $\tau \sim 100$.
   g. Using the equations in Ch. 7 in KW, Estimate the velocity of convection cells. What is their Mach number?
   h. Compute ∇. Is the convection efficient or inefficient?

   i. Estimate the turnover time of the eddies and compare with the $\sim$ 100-sec g-mode period.

4. *Diffusion of Elements.* Given that the abundance of elements is important for both nuclear burning rates and for atmospheric abundances, it is good to know if there is time during stellar evolution for heavy elements to sink to the center of a star.
   a. What is the macroscopic force on an ion with charge Ze and mass Am_u in a star otherwise composed of pure hydrogen? (Don't forget the electric field $eE = m_p g/2$ that has to be present to keep the protons from sinking and the electrons from flying out of the star.)
   b. Roughly how often does an ion collide with the electrons and protons in the gas? Which collisions determine the rate of downward settling? Estimate numerical values for ions near the half-mass radius of a $1\,M_\odot$ main-sequence (pure hydrogen) star.
   c. Combining (a) and (b), calculate the rate of downward drift of an ion as a function of the density, temperature, and gravity in the gas. How does the time to drift a radius R scale with the internal temperature of the star (scale to a fiducial value of 10^7 K)? How does this timescale for a typical ion (i.e., to settle from the half-mass radius to the center) compare to the main-sequence lifetime as a function of the mass of the star?
   d. Check to see if there is time for ions to settle at the photosphere for stars more massive than the Sun. Using the calculation of part (c), how long will it take an ion to sink a scale height at the photosphere? (Assume that $P_{\rm ph} = g/\kappa_T$ where κ_T is the Thomson opacity. Then use the effective temperature $T_{\rm eff}$ for a main-sequence star to calculate $t_{\rm drift}$. Should we see heavy elements in the photosphere? or just pure hydrogen?
   e. Now do the same as (d) but for a $0.6\,M_\odot$ white dwarf with $T_{\rm eff} = 20,000$ K.