Ay123

Fall 2005

STELLAR STRUCTURE AND EVOLUTION

Problem Set 5

Due Tuesday, November 1, 2005

1. To do this problem, you will need to refer to the Caughlan-Fowler 1988 (CN1988) nuclear reaction rates available at `http://www.phy.ornl.gov/astrophysics/data/data.html`. In novae (accreting white dwarfs) and X-ray bursters (accreting neutron stars), hydrogen burns, often explosively and at much higher temperature than in main-sequence stars. Use the CN88 rates to do each of the following steps for hydrogen densities ρ with $\log_{10}\rho = -2, 0, 2, 4, 6$:
   a. In main-sequence stars, the CNO cycle proceeds as ${}^{12}C(p,\gamma){}^{13}N(e^{+}\nu_e){}^{13}C(p,\gamma){}^{14}N$, etc. Find as a function of ρ (for values given above) the temperature above which this part of the chain will be replaced by the hot CNO (HCNO) chain: ${}^{12}C(p,\gamma){}^{13}N(p,\gamma)$ ${}^{14}O(e^{+}\nu_e){}^{14}N$ (the rest of the chain is the same as the ordinary CNO cycle). *
   b. At the temperature of main-sequence stars, the rate of energy generation ϵ_{CNO} by the CNO cycle increases rapidly with temperature. Find, as a function of density (for values given above) the temperature above which ϵ_{HCNO} will saturate at a maximum value. (Note: Saturation occurs when the rate-limiting step is a beta decay.)
   c. Show that the maximum value is is $\epsilon_{\max} = 5.9\times10^{15}\, Z_{CNO}$ erg g^{-1} s^{-1}, where Z_{CNO} is the mass fraction in CNO isotopes participating in the HCNO cycle ($Z_{CNO} \sim 0.006$ for solar abundances). Hint: neutrinos from the positron decay of ${}^{14}O$ and ${}^{15}O$ remove on average 2.03 MeV.

2. In problem 1, you assumed that the decay rates of the oxygen isotopes were given by their vacuum e^{+} decay rates. Above what hydrogen density would you expect decay rates to be significantly modified by e^{-} capture of free electrons? How will this affect $\epsilon_{\max}$?

3. If accretion of hydrogen onto a neutron star keeps the hydrogen hot enough that it burns at the rate $\epsilon_{\max}$ that you found in problem 1, calculate the time it takes the hydrogen (plus a solar admixture of CNO) to burn. If the accretion rate is $10^{-10}\, M_\odot$ yr^{-1}, what is the total mass of the layer on the surface of the neutron star

* Note that the notation ${}^{13}N(e^{+}\nu_e){}^{13}C$ implies ${}^{13}N \rightarrow {}^{13}C + e^{+} + \nu_e$.

which is burning hydrogen, and what is its surface density (in g cm^{-2})? Assume the radius of the neutron star to be 10 km.

4. Assume a neutron star of mass $1.4\,M_\odot$. Pretend Newtonian gravity is correct (an assumption which is still good to ~ 2 here). Use the equation of hydrostatic equilibrium and the equation of state for degenerate electrons to derive properties of the surface layers of the neutron star:

a. Show that the column density above any layer is proportional to the pressure in the layer. Then use your results from problem 4 to estimate the (electron) pressure in a neutron star accreting at $10^{-10}\,M_\odot\ \mathrm{yr}^{-1}$.

b. Below what temperatures will your assumption that the pressure in this layer is dominated by degenerate electrons be valid?

c. Estimate the depth (in cm) below the surface of this layer.

d. Estimate the time it will take photons to escape from that depth. Assume the temperature is high enough that all electrons are unbound, so the opacity just that of free electrons.