Ay123

Fall 2005

STELLAR STRUCTURE AND EVOLUTION

Problem Set 6

Due Tuesday, November 8, 2005

1. Verify that the following variant of the Runge-Kutta method to solve the equation $y' = f(x,y)$ is also accurate to 4th order in each step.

$$k_1 = h * f[x, y]$$

$$k_2 = h * f[x + h/2, y + k_1/2]$$

$$k_3 = h * f[x + h/2, y + k_1 * (\sqrt{2} - 1)/2 + k_2 * (1 - \sqrt{2}/2)]$$

$$k_4 = h * f[x + h, y - \sqrt{2}/2 * k_2 + (1 + \sqrt{2}/2) * k_3]$$

$$y(x+h) = y(x) + \left[k_1 + (2 - \sqrt{2}) * k_2 + (2 + \sqrt{2}) * k_3 + k_4\right]/6$$

2. Write you own code using the above method. Use it to solve the equation $y'(x) = y(x)$, with $y(0) = 1$. Use the following value of h $1, 0.1, 0.01, 0.001$, and plot the difference between the numerical value of $y(1)$ and the analytical solution $y(1) = e$. Show that the error is proportional to $O(h^4)$ as expected. What is the coefficient of proportionality? Is that roughly what you expected?

3. Use your code and solve the Lane-Emden equation (you will have to generalize it to a two component vector y). Once your solution crosses zero (θ becomes negative), use linear interpolation between the last two points to find a reasonable estimate for ξ_1. Reconstruct table 7.1 of HKT and provide plots of the density profiles.