General Relativity Ph236b
Problem Set 1
Due: In class, January 10, 2005

1. Show that the two forms of the Friedmann equation lead to identical dynamics. Do *not* assume that the matter is only pressureless.

2. The point of this problem is to show that the Friedmann equations for a Universe consisting of nonrelativistic matter can be derived with a Newtonian analysis. To do so, you consider an infinite homogeneous universe and then draw a sphere of radius a around an arbitrary point. You then argue that the universe outside the sphere can be considered as a sequence of concentric spherical shells and therefore do not impact the dynamics of the sphere. The dynamics of this universe then reduce to the dynamics of an expanding homogeneous sphere. Show that the equation of motion for the radius a of a homogeneous sphere are the Friedmann equations (both forms). Once you've done that, consider whether the same analysis can be used for relativistic matter by replacing the gravitating mass density ρ by $\rho + 3p$, where p is the pressure.

3. Find the coordinate transformation that shows that the Milne spacetime (i.e., the FRW spacetime with Friedmann equation $H^2 \propto a^{-2}$) is equivalent to a Minkowski spacetime. Explain what is going on.

4. Derive the photon redshift and the velocity decay for a massive particle in a FRW spacetime from the geodesic equation or by finding a conserved quantity associated with a Killing vector.

5. The age t_0 of the Universe in the standard cosmological model depends on the current value of the Hubble parameter, $H_0 = 100\,h$ km/sec/Mpc, as well as on Ω_0, the current density in units of the critical density. In class, we showed that if $\Omega_m = 1$ and $\Omega_\Lambda = 0$, then $t_0(h) = 6.7\,h^{-1}$ Gyr. (a) Your assignment is to generalize this result and derive an expression for the age of the Universe for $\Omega_m > 1$ and for $\Omega_m < 1$, both for $\Omega_\Lambda = 0$. (This shouldn't be too tricky—the answers are in the books. But still, you should derive the equations yourself.) Then, plot contours for $t_0 =$10 Gyr, 13 Gyr, and 17 Gyr on the $\Omega_m - h$. You may do this either by sketching the contours by hand, or you may generate such a plot with Mathematica, C, Fortran, or anything else, if you're so inclined. (b) Then, make the analogous plots, but for $\Omega_m + \Omega_\Lambda = 1$ (and restricting to $\Omega_m < 1$). (c) Stellar astrophysicists believe that the oldest stars are around 10–20 Gyr. If the correct value is somewhere around 14 Gyr, your plots should show you for which values of Ω_m and h there might be consistency.