General Relativity Ph236b
Problem Set 8
Due: In class, February 28, 2005

Suggested Reading: Dodelson 6.4–6.6; Liddle and Lyth, *Cosmological Inflation and Large-Scale Structure*, Ch. 7; Carroll, 8.8; Kolb and Turner, Ch. 8; Peebles, Ch. 17.

1. Using the formulas derived in class, calculate the rms gravitational-wave (GW) amplitude in the inflationary GW background in the LIGO/LISA frequency band (ask a friend or one of the many resident LIGO/LISA scientists or relativists here if you can't find the relevant frequencies rapidly in the literature). An order-of-magnitude calculation should be enough.

2. In a previous problem set, you considered inflation with a potential $V(\phi) = \lambda\phi^4$. You considered the homogeneous evolution of the scalar field, and the value of the scalar field at the end of inflation, and the number of e-foldings from some initial value ϕ_i. (a) Calculate the value of ϕ 60 e-folds before the end of inflation. This is presumably when the current observable horizon scale exits the horizon during inflation. Determine the values of the scalar and tensor spectral indices, n_s and n_t at this time. Estimate the curvature perturbation $\mathcal{R}$ at this time in terms of λ. What value of λ is required to get the right $\mathcal{R}$? What does this value of λ imply for the gravitational-wave amplitude h_k at the scale of the horizon today? To the best of your knowledge, is this model consistent with observations?

3. Using the relevant Einstein, continuity, and Euler-Lagrange equations, show that the curvature perturbation $\mathcal{R}$ remains constant when a given Fourier mode is well outside the horizon. Dodelson's book may be particularly useful for this problem. In fact, he more or less works it out, although in a different notation. If you can't at first work the problem out for yourself (it's not easy), then it may still be a good exercise to go through Dodelson's derivation and translate it into the notation and gauge choices we have used in class.