General Relativity Ph236b
Problem Set 9
Due: In class, March 7, 2005

Suggested Reading: Dodelson, Chs. 8 and 10; Carroll, 7.3 and 8.6. Cabella and Kamionkowski, astro-ph/0403392, Sections 8 and 9. Liddle and Lyth, *Cosmological Inflation and Large-Scale Structure*, Ch. 5. For gravitational lensing, see Blandford and Narayan, Annu. Rev. Astron. Astrophys. **30**, 311 (1992).

1. Do problem 7.3 in Carroll's book

2. Suppose that the Milky Way is embedded in a Galactic halo of the form considered in problem 6 of Problem Set 7. Now suppose that this dark halo was composed of neutron stars, each of mass $1.4\,M_\odot$, where $M_\odot = 2 \times 10^{33}$ erg is the mass of the Sun. Now suppose we look at a star in the Large Magellanic Cloud, a dwarf galaxy located at a distance 50 kpc from us, and we'll assume here that it is close to the Southern Galactic pole. If a halo neutron star passes near the line of sight to one of the stars in the Large Magellanic Cloud, then that star will be gravitationally lensed. (a) Estimate the Einstein radius (in arcsec) of the lens; you should find that this is very small compared with the resolution ($\sim$arcsec) of ground-based optical telescopes. If so, then the two images that appear when the source is strongly lensed will be unresolved. (b) Estimate the physical Einstein radius and compare it with the Schwarzchild radius. Does this result justify our use of the weak-field limit? (c) The "optical depth" for microlensing is the probability that the line of sight to a given star in the LMC falls within the Einstein ring of one the dark-halo neutron stars. Estimate this optical depth. The inverse of this optical depth is the number of stars in the LMC you would need to monitor if, on average, one star is strongly lensed at any given time. (d) Calculate the magnification of the two images during lensing, and plot it as a function of the impact parameter. (e) Of course, the lenses in the Galactic halo are moving, and so a source will not remain within the Einstein radius for more than a fixed amount of time. Estimate the time that a source spends within in the Einstein radius during a typical microlensing event. (f) Calculate and plot the brightness of a background star, as a function of time, as it passes behind the lens, for a minimum impact parameter b. Assume the relative angular velocity of lens and source is constant.

3. The quasars that are multiply imaged by gravitational lensing are usually lensed by elliptical galaxies. (a) Suppose we model the rotation curves of these elliptical galaxies as being approximately flat with rotation speed v_c (roughly 300 km/sec in the typical elliptical). Calculate the deflection angle for lensing. (b) Estimate (order of magnitude) the characteristic separation (be sure your result makes sense) of the sources, and then estimate (again, order of magnitude) the characteristic time delay between different sources. (c) Suppose that the fraction of critical density in elliptical galaxies is $\Omega_{\rm elliptical}$, and for this problem assume an Einstein-de Sitter Universe. Estimate the probability

that a high-redshift quasar is strongly lensed.

4. In class we wrote down a magnification tensor A for gravitational lensing. This tensor described the mapping from the image plane to the source plane (or *vice versa*), and it consisted of a convergence κ and two components, γ_1 and γ_2, of the shear. How would this tensor need to be modified if lensing *rotated* a source by an angle α? Is it possible for lensing to introduce a rotation?

5. The location of the first acoustic peak in the CMB temperature power spectrum depends on the angle subtended by the sound horizon at the surface of last scatter. (a) Set up a numerical calculation (this should be pretty simple with Mathematica) of the value $\ell_{\rm peak}$ corresponding to the sound horizon for a Universe (not necessarily flat) with a given Ω_m and Ω_Λ ($\Omega_m + \Omega_\Lambda = 1$ corresponding to flat). Make a contour plot in the Ω_m-Ω_Λ plane of equi-$\ell_{\rm peak}$ contours. For this part, you can assume that the sound speed before recombination is $1/\sqrt{3}$ times the speed of light. (b) How would the first acoustic peak move if the baryon density were higher? (c) How would the first acoustic peak move if the dark-energy equation-of-state parameter w differed from -1.